AUV Positioning Using Bathymetry Matching

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Abstract

A current research concern in AUV positioning is the constraint of INS error growth; approaches to this include surfacing for GPS fixes, terrain matching methods and acoustic transponder systems. This paper presents a positioning technique for AUV's that exploits existing bathymetric data in an operation area. Unlike many terrain matching approaches, which do positioning using distinct ocean bottom features, this method generates a position estimate by comparing the in-situ measured depth at the position of the AUV with available bathymetry data in the immediate area. This builds on contemporary AUV INS/VL navigation systems by incorporating a maximum likelihood estimate of position. Particular emphasis is placed on the design of the maximum likelihood estimator module which produces point-wise position estimates and typically contains a large error component with many outliers. This estimate is merged with the output of the AUV's INS/VL system which constrains the INS drift. Further position accuracy and faster convergence to the correct position can be achieved by incorporating a single slant range measurement from the AUV to a fixed location. The slant range is used as external constraint on both the INS and the MLE. This paper describes the implementation of this approach and the results of simulation studies.

Keywords

Navigation, AUV, terrain matching.

I. Introduction

ONE of the main problems facing the AUV community is the constraint of INS drift. The most common solution to this problem uses acoustic positioning systems. Long baseline systems require a series of transponders to be placed on the ocean floor and then the position of each determined. This system allows for localization only inside a limited area. For a more flexible area, short or ultra-short baseline acoustic positioning can be used. This requires a surface vessel to remain near the AUV at all times. Due to operational costs, it is desirable to reduce the amount of external equipment required for an AUV to operate independently.

Researchers have approached this problem differently. Lucido et al.[1], and Sistiaga et al.[2] match high resolution local depth maps against a large, low resolution reference map. Feder et al.[3], [4] use an on board multibeam sonar to map a region and use that map to navigate the area.

This paper presents a system that has been developed for use in areas where high resolution bathymetric data is available. External inputs required by the system are standard INS outputs, a single slant range to a known point and ocean depth measured at the present location of the AUV. The system is presented and results from computer simulations are shown.

II. Description of the System

This system consists of three modules: the terrain matching module, the state estimator, and the slant range corrector. The state estimator produces a predicted position based on dead reckoning or velocity input. This estimate is given to the terrain matching module which provides an updated estimate of location using measured ocean depth and detailed bathymetry in the neighborhood of the AUV. Given this updated estimate of position, the state estimator then updates the estimate of state. The slant range corrector takes the range from a transponder to the AUV and forces the estimated location to be that distance from the transponder. Each of these subsystems is described in detail below.

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A. Terrain Match

The terrain matching module of the system takes an estimated location of the AUV and ocean depth measured by the AUV at that position and uses a likelihood function to produce an updated location for the AUV.

Assuming that a method (dead reckoning, inertial navigation, etc.) exists to provide an estimated location of the AUV, a vector, $\hat{\boldsymbol{\alpha}} = (\hat{x}, \hat{y}, \hat{z})$, can be formed consisting of the assumed position $\hat{x}$, $\hat{y}$, and the measured ocean depth value $\hat{z}$. The $\hat{x}$ and $\hat{y}$ are eastings and northings in meters and $\hat{z}$ is the depth in meters. Each of these values has an associated measurement error.

This module requires a reasonably dense bathymetric data set in the neighborhood of the AUV. Each point in the bathymetric data set is a triple, $\boldsymbol{\alpha}_i = (x_i, y_i, z_i)$.

Let $\boldsymbol{\alpha}$ be the true location of the AUV, where $\boldsymbol{\alpha} = (x, y, z)$. The measurement error, $\mathbf{e}$, is defined as the difference between the true and estimated positions, $\mathbf{e} = \boldsymbol{\alpha} - \hat{\boldsymbol{\alpha}}$.

Viewed as an estimation problem, a maximum likelihood approach can be taken. That is, given observations from a distribution with unknown parameters the value of the observation that maximizes the likelihood function is the best estimate of the parameters. The $\hat{\boldsymbol{\alpha}}$ and $\boldsymbol{\alpha}$ are related through the likelihood function(LF) based on a known or assumed error distribution, f_e. The LF can be expressed as

$$L(\boldsymbol{\alpha}|\hat{\boldsymbol{\alpha}}) = f_e(\boldsymbol{\alpha} - \hat{\boldsymbol{\alpha}}) \tag{1}$$

Given the estimated position $\hat{\boldsymbol{\alpha}}$ and the bathymetry in the area $\{\boldsymbol{\alpha}_i\}$ the most likely location of the AUV can be determined as the $\boldsymbol{\alpha}_i$ which maximizes $L(\boldsymbol{\alpha}_i|\hat{\boldsymbol{\alpha}})$.

By further assuming that the errors between $\hat{\boldsymbol{\alpha}}$ and $\boldsymbol{\alpha}_i$ are only due to measurement and estimation errors, consistent with normal practice, the error $\mathbf{e}$ is assumed to be jointly Gaussian with zero mean and covariance $\boldsymbol{\Sigma}$. The Gaussian assumption is justified for the errors because they are likely to be small and equally likely to be positive or negative.

The LF can then be expressed as:

$$L(\boldsymbol{\alpha}_i|\hat{\boldsymbol{\alpha}}) = |2\pi\boldsymbol{\Sigma}|^{-1/2} \exp\left(-0.5(\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}})\right) \tag{2}$$

Maximizing this equation with respect to $\boldsymbol{\alpha}_i$ is readily shown to be equivalent to minimizing

$$\lambda(\boldsymbol{\alpha}_i|\hat{\boldsymbol{\alpha}}) = (\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}}) \tag{3}$$

Because $\boldsymbol{\Sigma}$ is a covariance matrix, it is positive semidefinite, and the likelihood function in (3) is strictly non-negative with minimum 0 at $\boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}}$. When evaluated over the bathymetric data set $\{\boldsymbol{\alpha}_i\}$, (3) produces a measure which can be viewed as the distance squared from the estimated position.

It has been observed in simulation that there are significant differences in the variances of the error in the along range versus cross range directions and little correlation between the error in these orthogonal directions. This corresponds to elliptical symmetry in the quadratic form of (3), with the major axis in the cross range direction and the minor axis along range. It is computationally advantageous to rotate the coordinate system as this has the effect of diagonalizing $\boldsymbol{\Sigma}$.

The position vectors $\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}}$ can be rotated so that the along range and cross range directions referenced to the transponder's location form the basis of the new coordinate system, with rotation matrix

$$\mathbf{R} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In the new coordinate system, (3) can be rewritten as:

$$\lambda = (\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}})^T \mathbf{R}^T \mathbf{R} \boldsymbol{\Sigma}^{-1} \mathbf{R}^T \mathbf{R}(\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}}) \tag{5}$$

Defining $\boldsymbol{\beta}_i$ as the rotated point $\mathbf{R}(\boldsymbol{\alpha}_i - \hat{\boldsymbol{\alpha}})$ and $\mathbf{R}\boldsymbol{\Sigma}\mathbf{R}^T$ as $\mathbf{C}$ yields a new LF in which the $\mathbf{C}$ is strictly diagonal due to the independence assumption in the new, rotated coordinate system. Rewriting (5) in terms of the new variables arrives at the final form of the LF.

$$\lambda = \boldsymbol{\beta}_i^T \mathbf{C} \boldsymbol{\beta}_i \tag{6}$$

The best estimate of the AUV's location is then the $\boldsymbol{\alpha}_i$ that minimizes (6).

B. State Estimator

The nature of the terrain matching does not take the dynamics of the AUV into account. By developing a state model for the system, these dynamics can be accounted for to improve the estimate of the AUV's position. The general approach taken is based on Kalman filter theory[5].

The problem is rotated into the same coordinate system that is used in the terrain matching module. Due to the presence of the ranging unit described in the next section, the along range location of the AUV can be measured to a high degree of accuracy. Therefore, the cross range dynamics are the only ones modeled. These dynamics were assumed to obey standard Newtonian motion.

$$\ddot{x}(t) = \frac{-F_c}{m}\dot{x}(t) + \frac{1}{m}u(t)$$

where F_c is the coefficient of friction, m the mass, $x(t)$ the cross range position of the AUV, and $u(t)$ is the instantaneous force applied to the AUV. Transforming to discrete time and setting $\mu = F_c T/m$ yields the difference equation

$$x_{n+1} = x_n + (1-\mu)(x_n - x_{n-1}) + \frac{T^2}{m}u_n$$

Taking a Kalman-like approach, the position of the system at time n is updated (using dead reckoning) to produce a prediction of the position at time $n+1$:

$$\hat{x}_{n+1}^{-} = x_n + (1-\mu)(x_n - x_{n-1})$$

This estimate of position is supplied to the MLE and compared with the output of the MLE, $\hat{x}_{n+1}$, to produce the error or innovations sequence:

$$\eta_{n+1} = \hat{x}_{n+1} - \hat{x}_{n+1}^{-}$$

The innovations sequence is in turn used to update the estimate of position,

$$x_{n+1} = \hat{x}_{n+1}^{-} + k\eta_{n+1}$$

The two parameters μ and k affect the optimality and rate of convergence of the system and are implementation specific.

The system can be made more stable by constraining the magnitude of the difference between two consecutive estimates by the nonlinearity

$$x_{n+1} = \begin{cases} x_n - \epsilon & x_{n+1} - x_n < -\epsilon \\ x_{n+1} & |x_{n+1} - x_n| \leq \epsilon \\ x_n + \epsilon & x_{n+1} - x_n > \epsilon \end{cases} \tag{7}$$

With ϵ being a system dependent tolerance that varies depending on vessel dynamics and sampling frequency.

C. Slant Range Corrector

The state estimator provides a means to account for the cross range position of the AUV, but another method is required to account for the along range location. By adding a transponder, the system can find the slant range to the known point accurately. This information, when combined with the depth of the AUV from on board sensors, allows the horizontal distance to be known accurately. However, it does not provide a measurement of angle.

This information is used to adjust the AUV's estimated position until the range is correct along a line between the transponder's location and the position estimate from the state estimator. As a result, the range information from the transponder serves to correct the along range position and the cross range position is taken from the state estimator.

III. Results

The results of the system simulations were excellent. The simulation studies demonstrate a relationship between the nature of the terrain and the system performance. The terrains tested generally could be classified as one of three types based on performance. The best terrains have prevailing contours that are close together and run parallel to the track of the AUV. If the contours remain close, but they are perpendicular to the direction of travel, performance is reduced. The worst performance occurs over terrain where there are few to no variations in the measured bathymetry.

Track line plots and position error plots have been provided for each test. The track line plots each contain three lines. The dotted line is the AUV's assumed track which is the best estimate of the AUV's location before any corrections are applied. The dashed line is the real track of the AUV, which was generated by the simulator. The solid line is the system's estimated track for the AUV. Startup transients are evident in each plot. Following is a plot of the error between the estimated and the real tracks. This error is shown in three parts: total error, the cross range component of the error, and the along range component of the error.

A. Performance over Different Terrains

The data set used in the simulations was collected in Pensacola bay and has an average horizontal sampling density of one sample per meter and a depth resolution of 6.5 cm. Over a large number of simulation runs over various terrains, the average RMS error in position estimated by this system was 0.9 meters. The actual RMS error of a given run is heavily dependent upon the terrain over which this system is operating.

The best terrain condition is characterized by a rough bottom that tends to have contours that extend parallel to the along range direction. A sample run over this terrain is shown in Figure 1. The standard deviation of the bottom depth along the AUV's track in this area was 3.3 meters. Figure 2 provides a breakdown of the error for this condition. As Figure 2 shows, most of the error is contained in the cross range estimation. This is expected because the ranging module provides an excellent estimate of the along range location. The terrain match works well under these conditions and provides the best cross range estimate of location. The range of RMS error values over this terrain type is less than 0.55 meters.

The next best terrain for this system is a rough bottom where the along range direction runs perpendicular to the contours. This condition penalizes the terrain match since the bottom does not change significantly over the search area. A sample run under this condition is shown in Figure 3. The standard deviation of the bottom depth along this track is 2.4 meters. The error plot is contained in Figure 4. The RMS error values range from 0.5 meters to 0.8 meters for this terrain.

The worst case terrain is nearly flat with a standard deviation along the AUV's track of 1.2 meters. The RMS error range of the estimated position is 0.8 to 1.2 meters. A representative run is shown in Figure 5, and the error is shown in Figure 6.

The terrain match performance is directly related to the amount of variation in the bottom depth, particularly in the cross range direction. This leads to the conclusion that cross range bottom roughness and slope can be used to dynamically determine system performance.

B. Performance in Extreme Conditions

There are two known conditions where the system has difficulty performing. Compensating for and identifying these conditions are subject to ongoing research.

One known adverse condition is when the AUV strays outside the coverage of the bathymetric data. This prevents the terrain match module from being able to chose the correct location. The system performs in an unpredictable manner in this situation. The system may swing out to the correct edge of the data. In this case, when the AUV hits the edge it will hang at the edge until the AUV reenters the area of bathymetric coverage. This case is detectable but there is no method to estimate where the AUV really is. Another possible result is that the system will find a most likely, but false, path within the bathymetric coverage area. This error is not detectable at present and it is not known if it can be compensated for.

In some oceanscapes, two or more different tracks may exist which produce the same depth profile as that measured by the AUV. In this case, the system follows the one closest to the current estimated location. In some cases the correct path can be deduced in post-processing by reversing the sense of time and running the AUV data through the algorithm backwards. If the results of running in both directions yield the same result, then that is the most likely solution. While it is not always possible to determine when a chosen path is incorrect, it is possible to determine when multiple paths exist by setting the initial position estimate to different locations and letting the system locate the most likely path while continuing to track the original path in parallel. If the new path does not converge to the original, then multiple paths exist.

IV. Conclusions

This paper presents a new method to locate AUV's that provides accuracies comparable to current approaches with significantly less hardware. The approach uses ocean depth measured by the AUV, slant range to a known transponder location and a high resolution bathymetric data set to produce an estimate of AUV's position with accuracy approaching the sampling density of the bathymetric data set. The results have been shown to depend on the terrain over which the AUV is operating.

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References

[1] L. Lucido, J. Opderbecke, V. Rigaud, R. Deriche, and Z. Zhang, "A terrain referenced underwater positioning using sonar bathymetric profiles and multiscale analysis," *Oceans, 1996*, 1996.

[2] M. Sistiaga, J. Opderbecke, M.J. Aldon, and V. Rigaud, "Map based underwater navigation using a multibeam echosounder," *Oceans, 1998*, 1998.

[3] H. Feder, J.L. Leonard, and C.M. Smith, "Adaptive mapping and navigation using sonar," MIT Marine Robotics Laboratory Technical Memorandum, 1998.

[4] H. Feder, J.L. Leonard, C.M. Smith, and A. Bennett, "Autonomous underwater vehicle navigation," MIT Marine Robotics Laboratory Technical Memorandum, 1998.

[5] Robert Grover Brown and Patrick Y.C. Hwang, *Introduction to Random Signals and applied Kalman Filtering*, John Wiley and Sons, 1997.

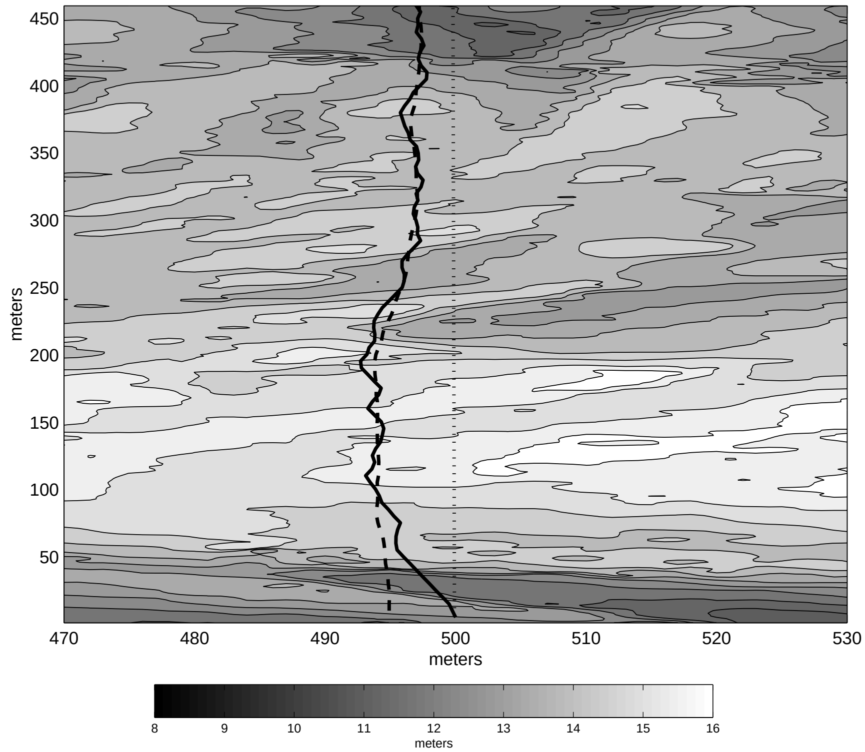

Fig. 1. Best Operating Conditions
Solid Line - Estimated Course
Dashed Line - Real Course
Dotted Line - Best Estimate Before Correction

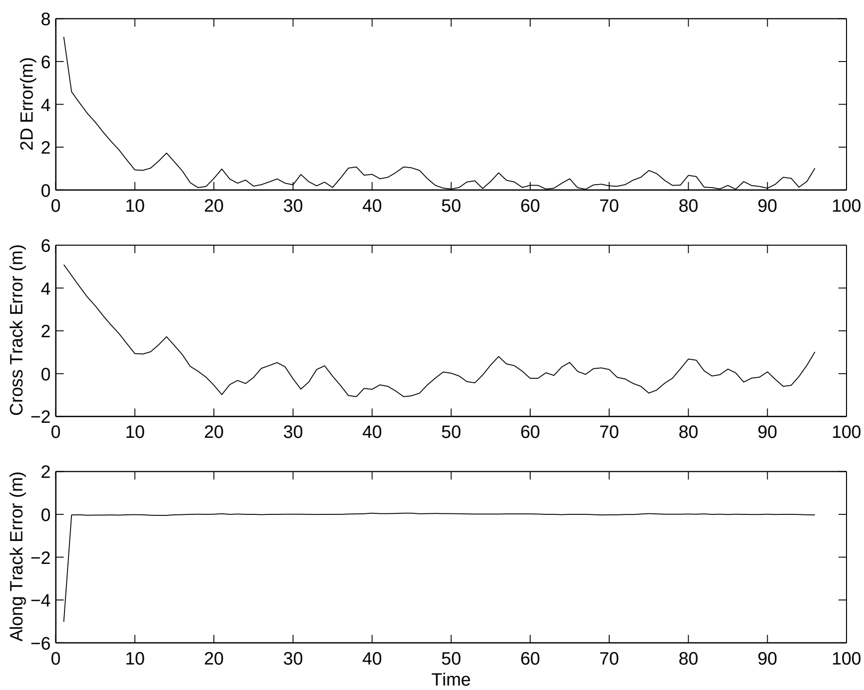

Fig. 2. Error Plots for Best Operating Conditions

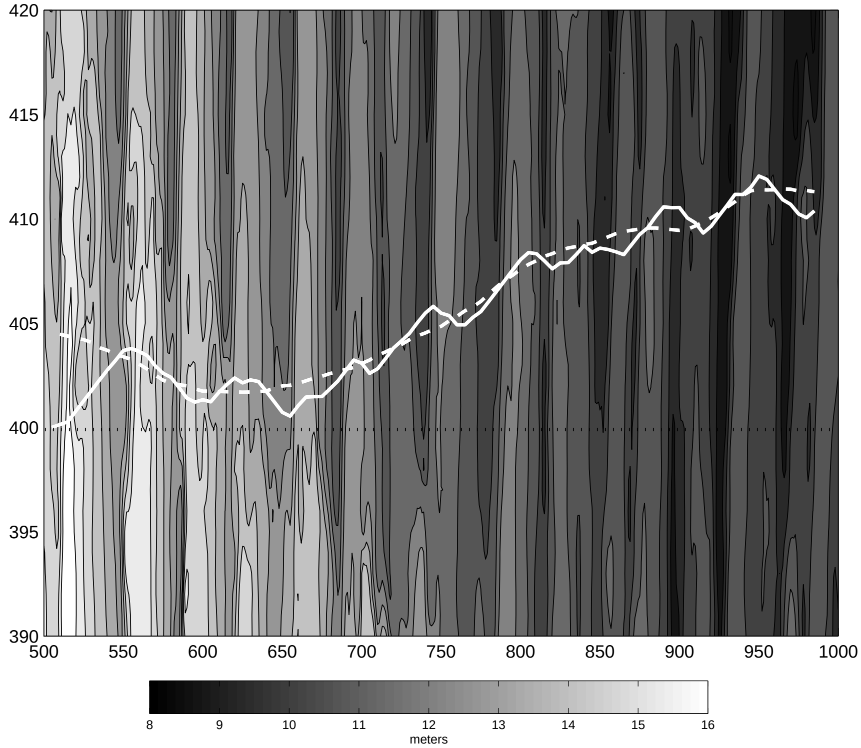

Fig. 3. Mid case Operating Conditions
Solid Line – Estimated Course
Dashed Line – Real Course
Dotted Line – Best Estimate Before Correction

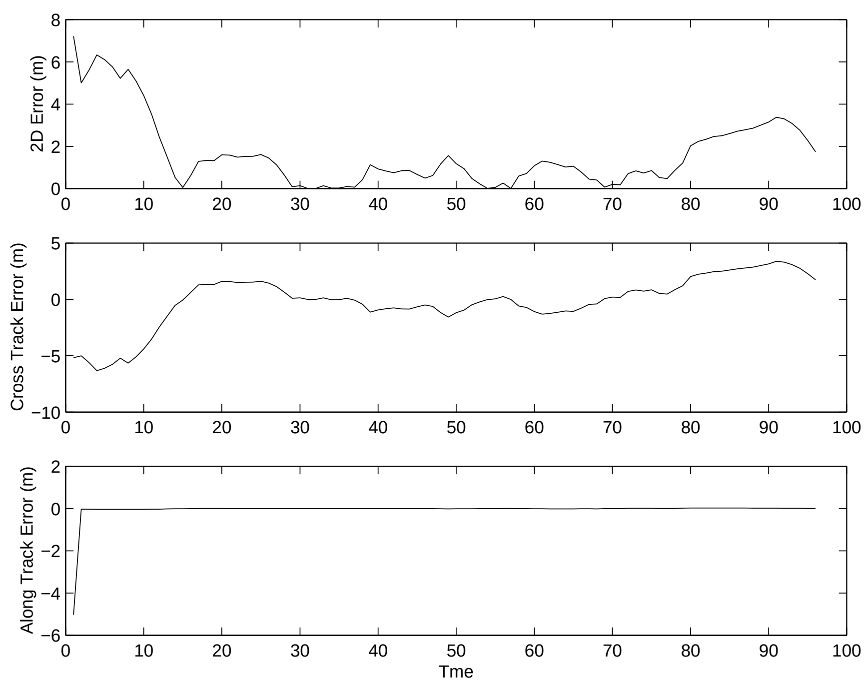

Fig. 4. Error Plots for Mid case Operating Conditions

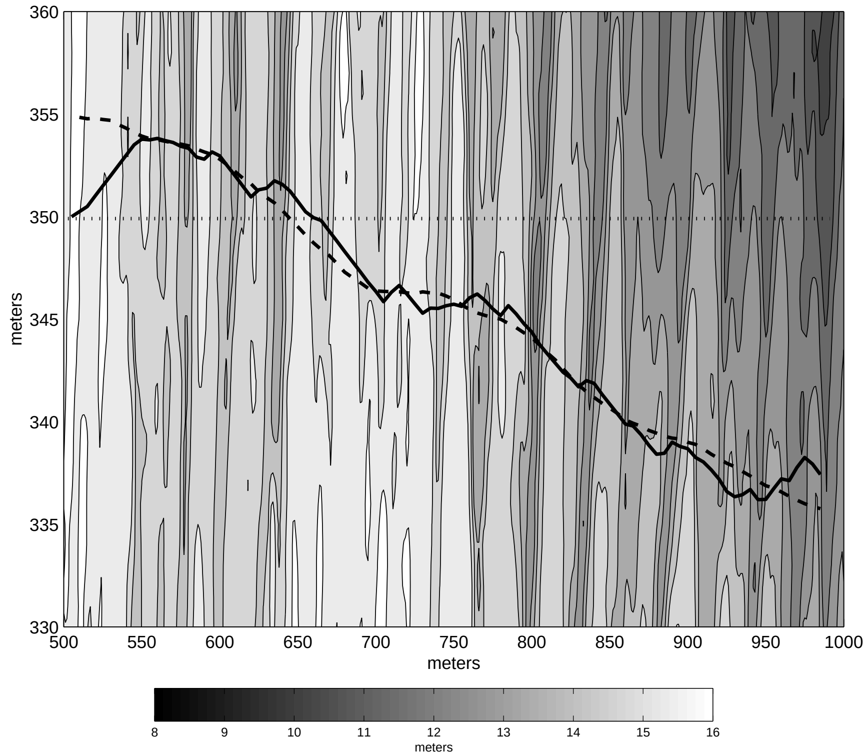

Fig. 5. Marginal Operating Conditions
Solid Line – Estimated Course
Dashed Line – Real Course
Dotted Line – Best Estimate Before Correction

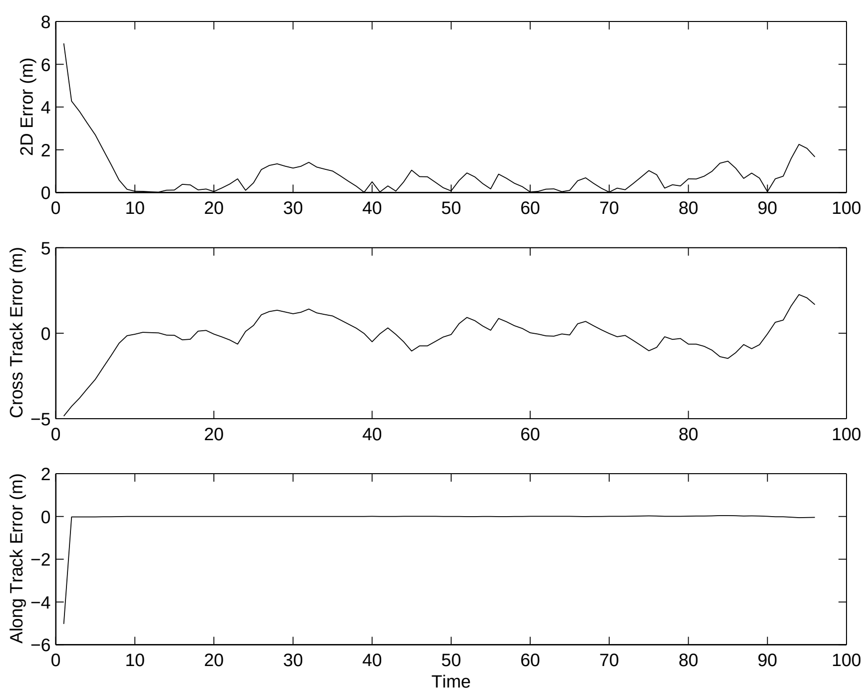

Fig. 6. Error Plots for Marginal Operating Conditions